

- (b) For some integer $n \geq 1$, let f have a continuous n^{th} derivative in the open interval (a, b) . Suppose also that for some interior point C in (a, b) we have $f'(c) = f''(c) = \dots = f^{(n-1)}(c) = 0$, but $f^{(n)}(c) \neq 0$. Then for n even, f has a local minimum at c if $f^{(n)}(c) > 0$, and a local maximum at c if $f^{(n)}(c) < 0$. If n is odd, there is neither a local maximum nor a local minimum at c .

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Let C be a constant and f and g be two measurable functions defined on the same domain then the functions $f + c, cf, f + g, f - g$ and fg are also measurable.
17. State and prove Fatou's Lemma.
18. State and prove Jordan's Theorem.
19. State and prove Chain Rule.
20. State and prove Inverse Function Theorem.

NOVEMBER/DECEMBER 2025

23PMA22 — REAL ANALYSIS – II

Time : Three hours

Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define Lebesgue outer measure.
2. Define Measurable sets.
3. Define Integrable of a Measurable function.
4. Define Lebesgue Integral.
5. Define Orthogonal system of functions.
6. State Dini's test.
7. Define Linear functions.
8. Write the matrix form of the chain rule.
9. Define saddle point.
10. State Jacobian Determinant.



SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) If E_1 and E_2 are measurable, then to show that $E_1 \cup E_2$ is also measurable.

Or

- (b) Prove that the interval (α, ∞) is measurable.

12. (a) Let ϕ and ψ be simple functions which vanish outside a set of finite measure. Then prove that $\int (\alpha\phi + b\psi) = \alpha \int \phi + b \int \psi$, and if $\phi \geq \psi$ a.e., then, prove that $\int \phi \geq \int \psi$.

Or

- (b) State and prove Lebesgue Convergence Theorem.

13. (a) Assume that $g(o^+)$ exists and suppose that for some $\delta > 0$, the Lebesgue integral $\int_0^\delta \frac{g(+)-g(o^+)}{t} dt$ exists, then we have
- $$\lim_{\alpha \rightarrow \infty} \frac{2}{\pi} \int_0^\delta g(t) \frac{\sin \alpha t}{t} dt = g(o^+).$$

Or

2

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- (b) Let f be a real-valued and continuous on a compact interval $[a, b]$. Then for every $\epsilon > 0$, there is a polynomial P (which may depend on E) such that $|f(x) - p(x)| < \epsilon$ for every x in $[a, b]$.

14. (a) If f is differentiable at c , then f is continuous at c .

Or

- (b) Let f and $D_2 f$ be continuous on a rectangle $[a, b] \times [c, d]$. Let p and q be differentiable on $[c, d]$, where $p(y) \in [a, b]$ $q(y) \in [a, b]$ for each y in $[c, d]$. Define F by the equation.

$$F(y) = \int_{p(y)}^{q(y)} f(x, y) dx, \text{ if } y \in [c, d]. \text{ Then } F'(y)$$

exists for each y in (c, d) and is given by

$$F'(y) = \int_{p(y)}^{q(y)} D_2 f(x; y) dx + f(q(y), y)q'(y) - f(p(y), y)p'(y)$$

15. (a) Let A be an open subset of R^n and assume that $f: A \rightarrow R^n$ has continuous partial derivatives $D_j f_i$ on A . If $F_f(x) \neq 0$ for all x in A , then f is an open mapping.

Or

3

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